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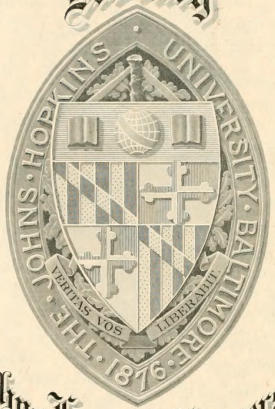

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The Action of the Asteroids on Mars

by

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Introduction.

Numerous papers have appeared, from time to time, on the size of individual members of the group of minor planets and the combined mass of the system they form about the sun. In this work, undertaken at the suggestion of Professor Newcomb as a subject for a thesis, I have endeavored to investigate the action of the asteroids on Mars.

To this purpose, I have divided the following paper into three parts;

Part first is an attempt to determine the probable superior limit which may be assigned to the mass of the asteroid ring.

Part second, is the computation of the "mean orbit" of the asteroids, and

Part third, is, from these data, to compute directly the annual variations in the elements of the orbit of Mars arising from the

secular terms in the perturbative function.
In what follows, I have shown that
the limiting value derived for the com
bined mass of the asteroids is far too
small to produce a motion, in one
year, of 0".0075 in the line of absides
of the orbit of Mars.

Section First.

The number of minor planets discovered up to November, 1894, was 404. The following table shows their position, with respect to the sun at the time of their discovery. The first column gives the number of hours of R.A. from the opposition point, and the second, the number of planets found within this angle.

Table I.

Hours.	Planets.
0 – 1	249
1 – 2	101
2 – 3	27
3 – 4	9
4 – 5	4
5 – 6	4
6 – 7	0
7 – 8	1

The positions of the remaining nine asteroids were not given with the original observations.
By comparing the apparent magnitudes at time of discovery with the mean opposition magnitudes given in the Jahrbuch, I have found, that for 72% of the minor planets, ~~they~~ were less.
Arranged in decades, in the order of discovery, and the means of the magnitudes taken for each decade, these mean magnitudes increase fairly regularly as follows,

Decade	Mag.	Dec.	Mag.	Dec.	Mag.
I	8.2	XI	11.0	XXI	11.5	XXXI	12.5
II	8.0	XII	11.2	XXII	11.1	XXXII	12.4
III	10.1	XIII	10.9	XXIII	11.9	XXXII	11.9
IV	9.1	XIV	11.1	XXIV	11.8	XXXIII	11.9
V	10.7	XV	11.6	XXV	11.9	XXXIV	11.7
VI	10.7	XVI	11.6	XXVI	12.7	XXXVI	11.8
VII	11.0	XVII	11.6	XXVII	12.0	XXXVII	11.2
VIII	11.1	XVIII	11.0	XXVIII	12.2	XXXVIII	12.1
IX	10.7	XIX	11.0	XXIX	12.2	XXXIX	11.6
X	10.9	XX	11.0	XXX	12.7	XL	11.6

*Berberich has shown the nearer planets were discovered first, and As the major axis of the "mean orbit" I find in the next section, is greater than that found by Srdstrup** in a similar manner, in 1884 this would indicate the same fact. In view of these considerations I think we may safely expect large additions to be made to the number of the minor planets already known.

In order to obtain an idea of the combined mass of the 311 asteroids whose orbits are given in the "Jahrbuch" for 1894, I have reduced the mean opposition magnitudes to the apparent magnitude at a distance 2.56 from the sun. This distance is near the mean of the distances of the individual members of the group, and I have chosen it because it satisfies the equation

$$5(\log 6 + \log(6-1)) = 3.$$

To reduce the magnitude of any asteroid, then,

* Ast. Nachr. No. 2827.
** Ast. Nachr. No 2740.

to the magnitude it would have if seen in mean opposition at a distance $(6-1)_M$ from earth. It is only necessary to add 3 to the quantity, g, given for each planet in the Jahrbuch.

~~When~~ If V_1 and V_2, m and m_1, are respectively the volumes and mean opposition magnitudes of two planets at the same distance from the sun, we easily find, assuming the albedos to be the same,

$$\frac{V_1}{V_2} = \left(\frac{10}{4}\right)^{\frac{3}{2}(m-m_1)}$$

or $$\log V_1 = \log V + .6(m-m_1)$$

In this way I have expressed the volumes of 310 asteroids in terms of the volumes of Vesta, and have computed the following table, which gives at once the relative volumes of any asteroid, by entering it with the quantity

$$M = g + 3.$$

Table III

$\overline{V_1} - \overline{V}\,\delta$

M	log δ	No. Asteroids	M	log δ	No Asteroids
7	9.00	2	10.1	8.14	10
7.5	9.70	1	10.2	8.08	8
8.4	9.16	2	10.3	8.02	12
8.5	9.10	1	10.4	7.96	13
8.8	8.92	2	10.5	7.90	10
8.9	8.86	1	10.6	7.84	7
9.0	8.80	1	10.7	7.78	11
9.1	8.74	2	10.8	7.72	9
9.2	8.68	1	10.9	7.66	6
9.3	8.62	2	11.0	7.60	11
9.4	8.56	1	11.1	7.54	8
9.5	8.50	5	11.2	7.48	14
9.6	8.44	5	11.3	7.42	12
9.7	8.38	6	11.4	7.36	10
9.8	8.32	4	11.5	7.30	10
9.9	8.26	4	11.6	7.24	9
10.0	8.20	6	11.7	7.18	6

Table III

M	log δ	Astrid	M	log δ	Astrids
11.8	7.12	10	13.5	6.11	--
11.9	7.06	6	13.6	6.06	1
12.0	7.00	7	13.7	6.00	—
12.1	6.94	6	13.8	5.94	1
12.2	6.88	7	13.9	5.88	1
12.3	6.82	5	14.0	5.82	2
12.4	6.76	8	14.1	5.76	4
12.5	6.70	7	14.2	5.70	1
12.6	6.64	5	14.3	5.64	2
12.7	6.58	3	14.4	5.58	1
12.8	6.52	1	14.5	5.52	1
12.9	6.46	1	14.6	5.46	1
13.0	6.40	1	14.7	5.40	2
13.1	6.34	2	14.8	5.34	--
13.2	6.28	3	14.9	5.28	--
13.3	6.22	—	15.0	5.22	--
13.4	6.16	4	15.2	5.10	1
			15.6	4.86	1

From this table I have found,

Volume of 311 Asteroids 8.181 Vesta.

Omitting Ceres Pallas and Vesta the volume is reduced to = 2.684 times the volume of Vesta.

Professor Barnard's measures of the diameters of Ceres Pallas and Vesta would give the albedoes of these three bodies, but as there are no measures of the greate mass of the minor planets, and as in every case, except these three, we are forced to estimate the relative volumes as I have done it, I have not thought it worth while to proceed differently in a few special cases.

Professor Barnards measures change the formerly accepted order of magnitude of Vesta, Ceres, Pallas, to Ceres, Pallas, Vesta, showing that the albedo is not a constant quantity, and relatively large in the case

of Vesta. Roughly, then, the volume of the entire known asteroid belt is about twice as great as the combined volumes of the three largest members.

To obtain some idea of the mass, I have assumed the density constant also, and equal to the density of Mars. The relative densities of the major planets seem to obey no apparent law which would enable us to interpolate the density of the minor planets, and Mars being nearest in size and position of the members of the solar system it seems natural as a rough first approximation to choose its density and albedo for the density and albedo of the asteroids.

There is a little more evidence in the case of the albedo, however. Müller* finds that the light curves of several members of the

* Potsdam - Publ. des Astro. Phys Obs.

asteroids studied by him are very similar to those of Mercury and Mars, and he computes the diameters of a number of the planets, assuming the albedo is equal to, first, to the albedo of Mercury, and, then, to that of Mars.

Estimates of the diameters of individual members of the asteroid group have been made by Barnard[x], Pickering[xx], Müller, Jacchini and Mellowick[xxx], Argelander and Stone

I have not used the earlier determinations of Argelander and Stone, as they differ very materially from those made with more accurate instruments. Jacchini and Mellowick give a diameter for Vesta which agrees closely with that given by Müller on the hypothesis that the albedo of Vesta is equal to the albedo of Mercury.

As the relative volumes are functions of the

x (Monthly Notices R.A.S. Vol. LIV No. 9)
xx (H.C.O. Annals Vol XI)
xxx Astr. Nachr. No 2363

differences of the magnitudes, the resulting total volume of the asteroids would depend upon the choice of the planet in terms of whose volume the volumes of all the rest are expressed, unless the relative magnitudes were similarly estimated by the various observers. I have chosen Vesta, the largest planet in common observed by Barnard, Pickering and Müller.

Reducing the the determinations of the diameter of Vesta to values corresponding to the magnitude of Vesta given in the Jahrbuch, the photometric determinations of Pickering and Müller and the direct measures of Barnard give the following:

Volume of Vesta in terms of Volume of Mars,

(1) from Pickering's estimate of the diameter of Vesta, assuming the albedo = albedo of Mars ..... = 000022

(2) from Müller's, (1) estimate of the diameter of Vesta

assuming the albedo = albedo of Mars 0.00065

(3) from Müller's estimate, assuming
the albedo = albedo of Mercury 0.00129

(4) from Barnard's measures of diameter of Vesta 0.00018

Combined Mass of 311 Asteroids, in terms of the
Mass of Mars,

(1) from Pickering's estimate as above 0.00112

" Müller " " " " (1) 0.00336

" " " " " (2) 0.00666

" Barnard's measures 0.00293

In the case of Barnard's measures I com-
puted the mass of Ceres and Pallas sepa-
rately and added it to the mass of
the remaining 309, and I took 4230 miles
for the mean radius of Mars
These numbers give us an idea of what
the probable mass of the known members
of the group. To determine the probable limit
to be assigned to the entire mass including

the planets
which are likely to be discovered I have proceeded as follows.

If the asteroids are arranged in groups, all those below the 9th magnitude (in table III) forming the first group, those between the 9th and 10th magnitudes forming the second, and so on, the following numbers are obtained,

9th	10th	11th	12th	13th	14th
9	31	94	93	141	16

For the 9th, 10th, and 11th magnitudes, these numbers very closely follow a geometrical progression where ratio is 3. This fact was pointed out by Professor Newcomb, in connexion with the table of Asteroids discovered at Nice*, but as I had independently noted it, I give my work here.

If the number of asteroids of a given magnitude increases in a geometrical progression, whose

Paper, by Perrotin - Bulletin. Astron. Tome XI. 1894
Comptes Rendus, " CXIX No. 2.

ratio is k, starting with the q^{th} magnitude the total mass of the planets of the m_1^{th} magnitude will be, (density and albedo, constant throughout),

$$\log. \text{mass} = (\log k - .6)\, m_1 + 5.4 - q \log k$$

where the unit mass is mass of asteroid of q^{th} magnitude. Proceeding by whole magnitudes of M_1, M_2 M_3 etc are respectively the combined masses of all asteroids of m_1^{th}, m_2^{th} etc magnitudes we can write,

$$M_1 = 10^{(\log k - .6) m_1} \cdot 10^{(5.4 - q \log k)}$$

$$M_2 = 10^{(\log k - .6)(m_1+1)} \cdot 10^{(5.4 - q \log k)}$$

$$\vdots = \text{- - - - - - - -}$$

$$M_n = 10^{(\log k - .6)(M+n-1)} \cdot 10^{(5.4 - q \log k)}$$

And the sum will be

$$\sum_{m_1}^{m_1+n-1} M = 10^{(5.4 - q \log k)} \cdot 10^{(\log k - .6) m_1} \left[1 + 10^{(\log k - .6)} + 10^{2(\log k - .6)} + 10^{3(\log k - .6)} + \text{- - -} \right].$$

This series will be convergent, so long as

$$\log k - .6 < 0.$$

The value,

$$k = 3.$$

makes, $\log k - .6 = -.123 +$

and the expression for the mass becomes

$$\sum_{10}^{\infty} M_i = \frac{3}{4} \cdot \left[\frac{1}{1 - 10^{-3.577}} \right]$$

$$= 3.04 \times \text{mass of asteroid of } 9^{\text{th}} \text{ mag.}$$

The combined mass of the asteroids, up to the 10th magnitude is (from Table III) very nearly ~~five~~ six times the mass of a tenth magnitude planet, hence the entire mass, on the assumptions, of constant albedo and density, and that the number of asteroids per magnitude increases in a geometrical progression, ratio 3, starting from the 10th mag.

is only about nine times the mass of an asteroid of the ninth magnitude, or approximately six times the mass of Vesta.

The series given on page 13 fails beyond the eleventh magnitude, but the later discoveries increase continually the number of planets whose magnitudes are relatively great.

On the other hand, it is not ~~supposable~~ probable that the numbers which represent the ~~known~~ actual number of asteroids of a given magnitudes follow exactly any law, and in the above I have endeavoured simply to obtain an idea of a superior limit to be given to the entire mass of the minor planets.

x

Section, Second

The researches of Professor Newcomb (On the Secular Variations and Mutual Relations of the Orbits of the Asteroids), and d'Arrest (Ueber das System der kleinen Planeten), have overthrown the hypotheses of Olbers as to the origin of the asteroids. The small planets have probably been formed by the condensation of an elliptic ring of matter. In a prize paper* presented to Royal Danish Acad., Svedstrup gives a method of finding the elements of this elliptic ring.

I have followed this method, and have recomputed most of the numbers formed by Svedstrup, in order to have the determination depend upon as many and as accurate orbits as possible. Changes in the mean orbit may also furnish evidence in regard to the addelicine to the known planets.

To find the elements of a fictitious planet which shall have the same secular effect as the asteroids, I have first found the six points of intersection of the orbit of each asteroid with six planes, passing through the sun and perpendicular to the plane of the ecliptic cutting the orbit of the earth in the points whose longitudes are respectively,

plane I, 30° ;
" II, 90° ,
" III, 150° ,
" IV, 210° ,
" V, 270 ,
" VI 330° .

At each point of intersection, I have imagined a mass placed proportional to the mass of the asteroid in question, and also to the time it takes the asteroid to describe an infinitesimal arc from that point.

Having found these points, then for each plane I have found the centre of gravity of the points in that plane.

In this part of the work I have used the elements of the orbits of the minor planets given in the Jahrbuch for 1894, omitting those planets which have been observed only in one opposition, and those which have been observed in more than one but lost later. I have also omitted the planets, the elements of whose orbits have not been corrected in the last thirty years.

In all there are (7) (8) (9) (12) (13) (15) (18) (20) (26) (27) (29) (32) (40) (58) (59) (69) (132) (155) (156) (157) (163) (175) (188) (193), (220), (274), (288), (290), (293), (296).

The last planet I used was (297). This made the orbit I have derived depend upon two hundred and sixty-seven minor planets, whose orbits are referred to the mean equator

and Equinox of 1890.0, and the Epoch and osculation, in only a few cases, being over three years from this date.

Points of intersection of the orbits with the planes.

To determine the rectilinear, heliocentric, coordinates of the various points of intersection of the orbits with the planes, let r, be the radius vector of one of these points, P, B its heliocentric latitude, and L the longitude of the plane which cuts by the orbit. In the spherical triangle, ΩPQ, where Ω and Q are respectively the node of the asteroidal orbit and Φ the point where a perpendicular from P intersects the ecliptic, we have, $QP = B$, $\Omega P = \omega + v$, $P\Omega Q = i$, and calling $\Omega PQ = N$, we can write,

$$\sin N \cos(\omega+v) = \cos i' \cos(L-\Omega),$$

$$\sin N \sin(\omega+v) = \sin(L-\Omega),$$

$$\sin N \sin B = \sin i' \sin(L-\Omega)$$

$$\sin N \cos B = \cos i'$$

These formulae give $\omega+v$ and B, then r' is found from

$$r = \frac{a\cos^2\varphi}{1+\sin\varphi\cos v},$$

The rectangular coordinates of a point of intersection then are
OX being directed toward a point longitude L, latitude $=0$
OY " " " " " " $=L+90°$ " $=0$
OZ " " " " " " " $=90°$

$$x = r\cos(\omega+v)\cos(L-\Omega) + r\sin(\omega+v)\sin(L-\Omega)\cos i' = r\cos B$$

$$y = -r\cos(\omega+v)\sin(L-\Omega) + r\sin(\omega+v)\cos(L-\Omega)\cos i' = 0$$

$$z = r\sin(\omega+v)\sin i' = r\sin B$$

Differentiating y

$$\frac{dy}{dt} = r\sin N \frac{dv}{dt},$$

since, $r^2\frac{dv}{dt} = k\sqrt{a}\cos\varphi$ we have,

$$\frac{d\eta}{dt} = \frac{k\sqrt{a}\cos\varphi \sin N}{r}$$

Suppose the masses of the planets spread out over the whole orbit, and taking

$$T = \frac{2\pi a^{3/2}}{k} = \text{unity}, \quad \text{we}$$

have,

$$\frac{dt}{T} = \frac{m r\, d\eta}{2\pi a^2 \cos\varphi \sin N}$$

Now the element of mass μ, will be inversely proportional to $\frac{d\eta}{dt}$ and directly proportional to m, so we can take

$$\mu = \frac{m r}{a^2 \cos\varphi \sin N}$$

$$= \frac{m x}{a^2 \cos\varphi \cos\iota} .$$

In Table III, gives the logarithms of m.* By this method I have found the following numbers

*In order to bring in the smaller planets in the computation, I have added 3 to logs in the table, which is equivalent to expressing all the masses in terms of the mass of an asteroid of 12th magnitude.

Plane I.

	u	μz	μx		v	v'	νx
1	397.433	− 154.839	106.482	18	———	———	———
2	193.982	− 193.109	46.994	19	4.922	+ 0.011	10.363
3	21.685	− 11.029	75.208	20	14.447	− 0.024	32.713
4	451.660	− 135.131	112.468	21	———	———	———
5	7.299	− 1.684	18.991	22	22.166	− 0.434	58.100
6	28.210	− 13.345	53.343	23	3.984	− 1.105	10.311
7	———	———	———	24	7.914	− 0.034	25.267
8	———	———	———	25	1.904	+ 0.126	4.821
9	———	———	———	26	———	———	———
10	61.294	+ 11.444	179.162	27	———	———	———
11	12.1411	− 2.364	29.211	28	1.944	− 0.024	26.870
12	———	———	———	29			
13	———	———	———	30	3.387	− 0.263	6.794
14	12.262	− 1.799	38.642	31	6.962	+ 0.259	1.690
15	———	———	———	32	———	———	———
16	21.736	− 2.563	98.185	33	0.307	+ 0.094	16.72
17	11.557	− 1.171	11.976	34	1.187	− 0.136	3.328

Plane 1

	u_1	u_2	u_x	u_1	u_2	u_x
28	0.672 +	0120	1.485 62	15093 -	5.777	111491
36	04185 +	0.160	0.922 53	.785 —	111	1.777
37	3.929 +	0.178	8.803 84	2.516 +	.236	6599
35	1.372 +	.1211	3.481 66	2.098 +	.285	6.886
39	20640 —	7.360	50.559 56	.670 —	065	1679
40	——	——		59 .7804 —	122	25.965
41	7.725 —	393	26.602 08	——	——	——
42	2.251 —	.575	4.802 59	——	——	——
43	2.243 —	.199	5.196 60	.785	-.083	1.679
44	5.512 —	.808	12.924 61	11.169 +	2.860	10.473
45	.7165 —	.124	12.130 64	785	-.095	1928
46	2.128 —	.090	11.912 63	.4053 +	925	11.575 ✓
47	2.644 +	.596	7.207 64	4.201 +	2.32	12.016
48	6.456 —	.940	14.493 65	11.232 —	1.944	41.002
49	1.092 +	.532	9.808 66	3.10 +	.114	.720
50	.568 —	.032	1.083 67	.785 —	.114	1.822
51	4.918 —	1.155	11.643 68	.1485 —	1.54	11.516

Plan 1

	u		u²	u³	u'		u'²	u'³
69	1.995	−	1,140	19.151	6,98	−	116	1.632
70	2.003	−	323	5.038	3.284	−	1.493	10.640
71	4.857	+	5.606	13.876	8.199	+	7.[illegible]	8.330
72	4.[illegible]	−	0.014	1.014	29 4.612	+	2.926	9.580
73	4.779	+	0.026	1.223	96 2.499	−	1.[illegible]	9.550
74	9.56[illegible]	−	0.[illegible]4	1.03[illegible]	01 4.328	+	1.[illegible]	10.391
75	0.[illegible]	+	0.062	1.434	9.[illegible]11	−	3.[illegible]	11.[illegible]
76	24.39[illegible]	+	0.016	9.[illegible]43.[illegible]	3.[illegible]19	+	6.[illegible]	10.134
77	1.[illegible]	−	0.[illegible]	3.[illegible]64.[illegible]	44.[illegible]	+	9.[illegible]	11.[illegible]
78	1.[illegible]83	−	0.[illegible]	3.[illegible]44	96 3.[illegible]16	+	9.[illegible]	9.[illegible]
79	2.[illegible]99	+	0.[illegible]49	7.[illegible]80	96 3.[illegible]56	+	2.[illegible]	10.[illegible]
80	1.11[illegible]	+	0.[illegible]08	2.411	94 2.[illegible]30	−	1.[illegible]	9.[illegible]
81	8.6[illegible]	+	1.[illegible]	1.[illegible]64	93 1.[illegible]99	+	2.[illegible]44	8.[illegible]
82	9.[illegible]99	+	0.008	2.[illegible]12	99		—	—
83	7.[illegible]11	+	0.008	2.030	100 1.[illegible]		0.[illegible]	11.[illegible]
84	4.[illegible]73	+	0.23	9.08	101 2.[illegible]		9.[illegible]	10.[illegible]
85	2.112	−	1.17[illegible]	5.[illegible]	102 1.[illegible]		0.[illegible]	8.[illegible]

Plane 1

	μ_1	$\mu\alpha_1$	μx_1	μ_2	$\mu\alpha_2$	μx_2
1	6503 −	1[illegible]	16592	2340 +	916	[illegible]
2	1113 −	0[illegible]	3091	2104 −	2130	[illegible]
3	1038 −	1[illegible]	2778	3919 −	193	[illegible]
4	[illegible] −	1[illegible]	9655	6944 +	181	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	3073 +	[illegible]	[illegible]	1979 −	2944	[illegible]
7	4224 +	0[illegible]	[illegible]	4999 +	007	[illegible]
8	4816 −	69[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	1138 +	2[illegible]	28555	3578 −	7444	9333
10	627 +	05[illegible]	[illegible]	9128 −	6445	[illegible]
11	10501 −	237	[illegible]	[illegible]	[illegible]	[illegible]
12	2101 −	36[illegible]	[illegible]	221 −	[illegible]	[illegible]
13	1524 +	[illegible]	[illegible]	—	—	—
14	4066 −	4224	[illegible]	3528 +	1522	[illegible]
15	2433 +	1307	[illegible]	[illegible]	[illegible]	[illegible]
16	1277 −	1131	[illegible]	1976 +	1244	[illegible]
17	2861 −	0744	[illegible]	4993 −	0744	[illegible]

Plate I.

Plane I

	μ_1	μ_2	ν_1	ν_2	ν_3	ν_4
1	1.336	-.045	49.1 138	—	—	—
2	2.057	+.601	11601 137	592	-.013	1.238
3	2.077	-.992	11115 136	556	-1.241	2.198
4	1.542	+.891	11732 101	838	-.341	2.184
5	—	—	— 102	1.590	+1.283	1.4128
6	1.310	-.222	3462 123	—	—	—
7	2.85	+.010	10011 111	3.217	-1.542	2.178
8	3.06	-.004	797 195	6.65	-.051	1.921
9	1.830	+.452	4938 196	12.379	-3.624	11.258
10	1.06	+.004	223 191	2.24	-.015	.547
11	2.625	-2.08	6808 198	8.8.	+2.10	12.165
12	2.13	-.067	1814 199	10.5	-.341	3.542
13	2.22	-1.71	370 200	2.223	+5.19	5.270
14	1.998	+.05	3341 201	5.8.	-.109	1.361
15	5.256	-4.264	11018 202	1.981	-2....	2.11135
16	4.21	.012	9011 203	1.02	+.053	2.333
17	1.814	1.62	5.315 204	-.38	-.121	1.875

Plane 1

	u	μ_2	μ_x		μ	μ_2	u_x
205	2600	- 0006	1333	222	1464	- 046	1599
206	1510	- 0048	1084	223	237	- 008	904
207	1216	+ 0011	1334	224	772	+ 128	3079
208	1993	+ 0011	2296	225	923	- 179	2913
209	21661	+ 0032	3783	226	1145	- 105	3835
210	1004	- 0043	2049	227	5911	+ 1310	2219
211	23221	+ 3132	6592	228	———	———	———
212	10201	+ 2023	23305	229	307	- 0015	9112
213	1990	- 2206	2996	230	24408	+ 4411	15399
214	2351	+ 01111	9000	231	631	+ 116	2117
215	2003	+ 0011	5117	232	067	- 018	1001
216	51711	+ 2556	10329	233	1222	+ 089	32011
217	456	- 0341	3112	234	3115	- 161	6205
218	14132	- 7116	11316	235	716	- 200	2100
219	4385	- 0249	9146	236	1397	- 173	2213
220				237	2111	- 0511	8376
221	33000	- 16365	92249	238	12801	- 3211	53834

Plane 1

239	039	−005		266		
240	203	−016				
241	2996	+ 676				
242	355	−007				
243	129	+ 006				
244	011	−000		261		
245				262		
246				263		
247				264		
248				265		
249				266		
250				267		
251	133			268		
252						
253						
254						
255						

Plan 1

283	1.252	−.211	10.33				
284	——	——	——	291	013	−.001	024
285	791	−.132	2.254	292	201	−.120	3.12
286	2.077	074	6.416	293			
287	1.84	+ .004	4.89	294	0.55	−.01	1.14
288	2.95	− .063	8.55	295	077	+ .008	1.83
289	.401	− .099	3.248	296	--		
381	0.35	+ .004	.098	297	260	1.057	3.41
381	.013	− .000	.124				
381	.034	− .011	.03				
382	1.571	+ .556	3.911				
382	.083	+ .008	.11				
383	——	——	——				
386	.314	− .269	.965				
387	1.3044	− .503	3.012				
388	.334	− .041	.128				
389	.319	− .046	.096				

Plane II

	u	u2	ux		u	u2	ux
1	[illegible]	+ 28613	932975 18				
2	144.390	− 17512	2553914 19		6.132	−0263	113441
3	37.692	− 17326	758388 20		144583	− 0333	3111 6
4	144222	− 36566	1496000 21			—	—
5	6.123	− 0976	138655 22		12378	+ 5781	59.219
6	32583	− 13838	696902 23		3287	+ 0.1163	6.816
7		—	— 24		7038	+ 0.225	10031
8	—	—	— 25		21114	− 2.1514	6.814
9	—	—	— 26		—	—	—
10	50865	+ 3.142	1142028 27		—	—	—
11	12763	− 1.670	357117 28		8.940	− 2813	20924
12	—	—	— 29				
13	—	—	— 30		2.484	+ 0.222	7.809
14	10610	+ 0.235	2668 31		5443	+ 5.186	12225
15	—	—	— 32		—	—	—
16	23.954	− 3077	650.533 33		1006	+ 0102	3.083
17	4824	− 0.964	13477 34		10601	− 0233	2.665

Plane II

	u	u z	u x		μ	μ z	μ x
35	6.981	+ 0202	2.0334	19	1.923	− 3.186	25.207
36	0.401	322	9.463	20	.711	− .104	1.441
37	3.840	+ 4.41	8.11404	21	3.082	+ 1.394	9.660
38	12.1	+ .158	2.920	22	3180	+ 1.030	8.226
39	22600	− 10.234	60.618	23	.827	− .350	2.653
40				24	9.178	− 6.454	35.221
41	6.922	− 0.69.5	20.113	25	—	—	—
42	208.5	+ .122	8.317	26	—	—	—
43	21.55	− 121	6.300	27	0.694	− 0.81	1.307
44	4.938	− 4.28	10.215	28	4.812	+ 4.135	13.982
45	11.01	− 116	11.798	29	832	− 0.30	2.212
46	2.444	− 2.80	6.253	30	4.92	+ 1.195	12.156
47	2.016	.832	9.270	31	4.028	+ 1.118	9.623
48	6.222	− 2.043	18.401	32	11.655	− 2.80	14.6..
49	4.446	+ .219	11.699	33	.329	+ 0.40	.711
50	.516	− 0.27	1.580	34	.9.4	− .2.3	2.6..
51	4.34	− 1.1.0	10.110	35	.5012	+ 1.646	16.6.3

Plane II

	u	μv	v^2	u	w	μt
103	1·68	− 1360	1956938	2311	+ 864	7·118
104	·111	1 112	30514	3031	+ 3312	11·109
105	5·12	− 672	3·19·1	2·84	− 314	12223
106	4·65	+ 1119	1141177	652	+ 110	1·1·6
107	3·191	− 4734	3·20·5	5634	− 803	1·3412
108	1·771	+ 655	8·61	20·3	− 1181	6113
109	·136	1 118	3·11	5·40	+ 064	13851
110	5·2·1	+ 832	1·1·635	11·11	+ 1512	12333
111	·1·18	+ 12·	·8·3	4215	+ 380	11030
112	·10	+ 066	1·1·1	0·50	− 4932	30·924
113	7·3	− 116	2·111	9·3·1	− 1·36	2·3·11
114	1·2·9	− 3·2	1·2·1	216	+ 0·2·	·2·5
115	1·322	1 ·01	·2·11	—	—	—
116	2·36	+ 2·13	9·012	3896	+ 1·24	12820
117	2·2·3	1 1·1·6	·2·11	1213	+ ·211	2·11
118	12·1	1 3·9	1·1·2	2·21	+ 3·2	1·11·9
119	·2032	− 5·18	·1·18	·9·2	− 9·1	1·1·1

Plane II

	u	μz	μx		μ	μz	ux
69	[illegible]	−2198	14803	86	750	+ 066	1 [illegible]
70	1364	+964	7015	87	3651	+ 619	1 [illegible]
71	4132	+ 4013	12 900	88	3 [illegible]	+ 1110	11 [illegible]
72	.456	− 114	1365	89	5541	1 2640	1 [illegible]
73	.417	+ 011	1 [illegible]	90	2886	+ 130	1 [illegible]
74	.890	− 160	2257	91	4170	+ [illegible]	[illegible]
75	933	+ 225	[illegible]	92	8033	[illegible]	[illegible]
76	2253	− 0193	6358	93	3515	+ 1 [illegible]	11 8243
77	1523	+ 153	3572	94	4107	+ 1 [illegible]	1 [illegible]
78	2531	+ 732	5349	95	3178	− 680	[illegible]
79	1 [illegible]	− 277	[illegible]	96	[illegible]	+ 1 [illegible]	8 161
80	1315	− 347	2924	97	2 [illegible]	− 1 [illegible]	[illegible]
81	[illegible]	+ 284	2079	98	876	+ [illegible]	1 [illegible]
82	1991	+ 081	1809	99	———	———	———
83	1 [illegible]	+ 137	1771	100	1775	− 456	1 [illegible]
84	[illegible]	+ 197	1 [illegible]	101	2867	+ 1332	1 [illegible]
85	[illegible]	− 1383	[illegible]	102	2 [illegible]	− 041	[illegible]

Mare II

	a	μ_2	μ_1		μ	μ_2	μ_1
177	1191	−029	3497	188			
178	2334	898	5746	189	535	−107	1297
179	2371	−1288	5796	190	5204	−1838	16786
180	1603	+ 942	5118	191	573	− 452	2371
181				192	9251	+2285	19834
182	1341	−1444	3630	193			
183	323	+ 019	774	194	4009	−3464	11134
184	322	+ 017	824	195	583	+ 196	1608
185	2051	−240	6069	196	13616	+ 1568	42863
186	002	+ 002	2121	197	366	+ 017	712
187	2381	−1540	5601	198	1076	− 010	2558
188	927	− 020	1873	199	1173	+ 007	4366
189	211	− 174	438	200	2282	+ 563	5560
190	930	+ 080	2152	201	671	− 178	1918
191	623	− 284	1389	202	1094	− 2343	3045
192	524	+ 201	1253	203	520	+ 132	2400
193	1633	+ 329	4701	204	671	− 272	2091

Plane II

	μ	μ2	μx	μ	μ2	μx
139	2241	−1726	7925 154	5132	11803	19887
138	387	+ 033	1013 155	———	———	———
137	3078	+ 1501	17861 156	———	———	———
110	1752	− 092	5603 157			———
111	1062	+ 1108	2538 158	502	+ 005	1176
112	225	+ 009	588 159	886	− 187	2436
113	281	+ 198	1054 160	793	+ 136	2031
114	2832	+ 133	6915 161	1068	+ 116	2784
115	1165	+ 130	2699 162	676	+ 149	1773
116	2382	+ 158	6966 163	———	———	———
117	122	− 024	2231 164	1035	+ 236	2415
118	2603	− 2260	5328 165	5501	+ 1615	18355
119	028	+ 001	057 166	252	− 078	570
120	2029	− 199	6026 167	208	− 028	613
121	665	+ 150	1608 168	3991	−910	13062
122	1203	+ 610	3139 169	597	+ 143	1483
123	3114	−1311	11244 170	564	+ 117	1349

Plane II

240	0113	- 015	1097	256	2306	- 165	699
240	208	- 007	4533	257	1136	+ 063	121
241	3085	038	9614	258	1420	- 181	3284
242	215	- 1110	793	259	1126	+ 024	4920
243	137	+ 006	3497	260	231	- 089	824
244	011	- 001	052	261	264	- 014	5305
245	580	067	1580	262	018	+ 004	036
246	946	- 125	2779	263	148	- 007	419
247	2053	+ 1887	4092	264	589	+ 179	1312
248	032	- 006	215	265	028	+ 031	071
249	021	+ 007	036	266	1017	- 313	2597
250	14114	1004	5277	267	056	+ 004	152
251	124	- 036	3565	268	643	- 014	1960
252	421	- 213	1311	269	206	- 059	652
253	079	- 035	219	270	526	- 011	125
254	021	+ 004	052	271	364	+ 016	10321
255	051	022	1352	272	098	- 013	213

Plane II

	u_1	u_2	u_4		u_1	u_2	u_4
203	.2140	−116	322	222	487	+ .011	1.739
201	.6049	−036	1600	223	218	+ .014	.596
207	.523	+ 030	518	224	787	+ .221	2.159
208	.1581	+ .0214	2220	225	1154	− [illegible]	4385
209	2.691	+ 1134	5934	226	1144	− .111	650
210	.293	+ .0411	1194	227	502	+ 126	1.586
211	3.234	+ .023	5254	228	——	——	——
212	1.013	+ .151	2815	230	330	+ 3141	1151
213	1107	− 221	3441	230	2006	− 438	5324
214	.3442	+ 054	9111	231	639	+ 2015	2246
215	210	+ 016	586	232	059	− 015	186
216	5639	−2415	12861	233	1336	− 259	3594
217	113	−122	6944	234	406	− 218	978
218	14726	−1063	3561	235	3411	+ 1113	12611
219	3791	−2561	14101	236	1622	− 5417	14383
220				237	257	1 011	692
221	.3664	−1731	11329	238	1308	−1923	3523

Plant II

	u	u2	u4		u	u2	u4
523	[illegible]	[illegible]	1295	[illegible]	—	—	
[illegible]	—	—	—	[illegible]	012	001	034
[illegible]	161	[illegible]	[illegible]	[illegible]	287	102	[illegible]
[illegible]	1384	[illegible]	5199	[illegible]	—	—	—
[illegible]	198	[illegible]	803	[illegible]	071	020	251
[illegible]	300	1 [illegible]	709	[illegible]	077	301	183
[illegible]	867	[illegible]	3923	[illegible]	—	—	—
[illegible]	033	011	[illegible]	[illegible]	300	121	1013
[illegible]	013	[illegible]	026				
[illegible]	026	[illegible]	0114				
[illegible]	1304	426	5149				
[illegible]	096	011	213				
[illegible]	—	—	—				
[illegible]	314	[illegible]	977				
[illegible]	134	[illegible]	[illegible]				
[illegible]	220	[illegible]	928				
[illegible]	456	185	1347				

Plane III

	u	u²	u⁴	u	u²	u⁴
1	335.331	+ 118.1111	84444116	1	———	———	———
2	169431	− 94478	35165619	6025	−0369	15566
3	47578	− 9885	1208332	15499	+ 03210	24005
4	431 310	+93256	1025764	21	———	———	———
5	5879	+ 0.173	12319	23780	+ 16232	66865
6	40.355	+ 564	109099	23123	+ 1166	64649
7	———	———	———	24	6708	+ 0153	18198
8	———	———	———	25	2423	− 2.371	66418
9	———	———	———	27	———	———	———
11	445443	− 6466	1388655	28	———	———	———
11	144244	+ 1044	38316	29	8944	+ 0231	2e902
12	———	———	———	26	———	———	———
13	———	———	———	30	4672	− 0133	100043
111	9198	2901	2006579	31	5870	+ 6229	144223
2	———	———	———	32	———	———	———
11	27170	− 0036	86230	32	1464	+ 0118	55346
12	44235	+ 0466	113411		1011	− 0132	24116

Plane III

	u	u_2	u_4	v	v_2	v_4
35	0.4141	+ 0.220	3.179	62	1.1324	+ 1.829	4.0443
36	0.693	+ 209	1.883	63	771	+ 017	1.734
37	1.1385	+ 264	1.9465	64	.2053	− 553	9.189
38	1.209	− 2.15	2.169	65	3.685	+ 922	11.106
39	2.5.4111	−1.708	96.825	66	829	− 511	2.452
40	———	———	———	67	1.201	− 6.602	31.702
41	5.273	− 171	12.420	68	———	———	———
42	3.089	1.211	9.058	69	———	———	———
43	2.142	− 3.97	4.930	60	734	− 063	1.517
44	5.135	+ 2.81	11.062	61	5.326	+ 0.477	19.820
45	2.812	+ 0.21	1.2.161	62	1022	+ 052	.2298
46	1.2.31	1.45	8.384	63	4.657	+ 164	11.744
47	3.111.7	+ .502	1.2.12	64	2.989	− 072	9.136
48	6.612	− 1.2.3	20.128	65	10.901	− 357	38.343
49	5.4.63	− 6.11	18.211	66	386	+ 034	1022
50	1.25	0.57	2.4.19	67	980	− 231	2.453
51	0.218	.212	.9.322	68	6.821	+ 2.983	22.103

Plate III

	u	u²	u⁴		u	u²	u⁴
60	[illegible]	1[illegible]6	15[illegible]90	86	9110	+ 232	31841
70	3[illegible]4	+ 1[illegible]92	6908	87	3808	+ 2647	14260
71	2832	− 980	9623	88	3843	− 866	12174
72	56[illegible]	− 114	1420	89	6631	− 1781	19548
73	4[illegible]05	+ 034	1325	90	2[illegible]9	+ 1103	10355
74	1136	− 189	3610	91	4443	+ 267	10928
75	1173	+ 178	4072	92	8375	− 3748	29177
76	2[illegible]29	− 234	7410	93	3606	+ 913	10581
77	1[illegible]02	+ 101	4462	94	4468	+ 1110	14281
78	2[illegible]40	+ 05[illegible]	5387	95	3609	− 2584	11282
79	2[illegible]25	− 3[illegible]1	5518	96	2720	− 260	[illegible]243
80	15[illegible]05	− [illegible]92	4192	97	3287	− 3[illegible]1	7988
81	1086	+ 229	3070	98	833	+ 207	1804
82	766	+ 069	1661	99	——	——	——
83	6[illegible]0	+ 112	1676	100	1986	+ 213	7020
84	[illegible]16	− [illegible]15	2080	101	3134	+ 387	9178
85	5[illegible]84	− 14[illegible]3	3682	102	291	− 068	8745

Plume III

	μ	μσ	μκ	μ	μσ	μκ
103	7268	+ 476	21031	120	2193	+ 178	11345
104	12111	+ 1854	38112	121	8938	+ 11335	33948
105	878	+ 1811	1989	122	3704	− 155	11566
106	4747	+ 1300	16119	123	723	− 178	1878
107	8689	− 2175	28584	124	5286	− 1115	13922
108	2579	+ 217	7450	125	2005	− 1511	5919
109	5418	+ 109	7369	126	598	+ 066	1600
110	5496	+ 1685	16068	127	4734	+ 1554	15166
111	1076	− 089	2585	128	4757	+ 1172	14009
112	775	− 010	2124	129	7658	+ 967	21059
113	913	+ 081	2028	130	12505	+ 1243	43033
114	1713	− 084	3952	131	201	+ 011	1113
115	2285	− 411	5150	132	—	—	—
116	3285	+ 488	7805	133	3465	− 206	10518
117	2704	+ 018	7945	134	1316	+ 192	8312
118	1834	+ 398	5983	135	2643	+ 075	7118
119	3270	− 721	8897	136	5181	− 134	13418

Plate III

	u	u²	μx	μ	μ²	μx
137	2.308	−.679	8176	1551	47 18	+ 1932	13353
138	1145	+ 066	1168	1585	—	—	—
139	6270	+ 1027	6289	156	—	—	—
140	1783	+ 2130	56111	1	—	—	—
141	1338	−.160	1128	158	536	−.020	1804
142	197	−.011	199	159	908	+ 012	3610
143	378	+ 012	910	160	8413	+ 098	2299
144	3564	+ 882	10965	161	11014	+ 352	29111
145	11411	+ 558	2602	162	6.72	+ 1.34	1.13
146	2184	+ 1207	5688	163	—	—	—
147	1411	−.077	2376	164	1365	+ 1801	11163
148	3264	+ 394	9743	165	536	−.150	1.1456
149	024	−.000	.064	166	.3111	+ .060	.8.5
150	2272	−.228	7557	167	2.9	−.00 1	6.7
151	6118	+ 170	1609	168	43603	−1045	1.51855
152	1.24	+ 822	4010	169	6.11	1 .6	1705
153	29415	−1712	12657	170	5.11	− 17.5	11120

Plane 1

	μ	$\mu\nu$	μx	ν	$\mu\nu$	νx
171	1118	+ 106	3137	188	——	——
172	2510	+ 039	6655	189 554	−100	1389
173	3000	+ 056	9278	190 5110	−906	18564
174	11459	−018	11238	191 975	−102	29.37
175		——	——	192 119919	+ 9111	33093
176	1614	−1705	.5258	193 ——	——	——
177	4110	+ 010	1253	194 11647	−810	14986
178	313	+ 026	782	195 5811	+ 121	1611
179	22148	−969	72885	196 135710	+ 5285	42574
180	091	− 001	208	197 283	+ 128	891
181	2671	+ 214	70111	198 1315	−551	3520
182	1089	+ 067	2583	199 1006	+ 768	.3213
183	360	+ 059	973	200 2595	−071	7176
184	881	+ 006	2183	201 934	−005	2461
185	68111	−572	20210	202 7171	+ 660	19850
186	573	+ 252	1536	203 976	+ 0118	2704
187	1293	+ 1119	2981	204 594	−193	1601

Plane III

	u	u2	ux		u	u1	ux
205	279	−130	779	222	4436	+ 0450	13491
206	624	+ 009	1677	223	2224	+ 021	626
207	217	+ 028	4[illegible]	224	7911	+ 085	20086
208	781	+ 014	2225	225	11446	−1318	45804
209	2577	+ 550	4142	226	166	+ 026	503
210	3[illegible]9	+ 075	910	227	1117	+ 003	1091
211	24[illegible]2	−429	7026	228	———	———	———
212	1117	−067	3456	229	3[illegible]3	+ 459	1388
213	1052	+ 169	3133	230	2632	−1067	6432
214	366	+ 012	952	231	612	+ 064	1814
215	216	+ 016	61[illegible]	232	[illegible]30	−006	112
216	7157	−4381	20724	233	11435	−531	41445
217	2440	−040	888	234	4911	+ 039	11404
218	1308	−326	3360	235	712	+ 330	2084
219	204	−295	1994	236	18[illegible]	−477	5997
220	———	———	———	237	21[illegible]	+ 098	623
221	2913	+ 324	13004	238	11449	−535	4326

Plate III

	u	u_7	u_8	u	u_7	u_8
239	054	− 010	1741	256 220	− 0511	6304
240	249	+ 0111	653	256 4175	+ 0358	1145
241	3295	− 495	10479	258 15-5	− 690	32101
242	311	− 133	1586	259 1264	+ 6413	3566
243	131	+ 000	372	260 2107	− 033	9446
244	012	− 001	039	261 326	+ 025	4178
245	106	+ 212	3338	262 021	+ 006	060
246	405	− 152	21811	263 160	− 010	4180
247	2681	+ 1625	6855	264 666	+ 358	19145
248	073	− 013	192	265 023	+ 002	0144
249	027	+ 001	073	266 1164	− 816	31133
250	2064	+ 1122	5922	269 0447	+ 0114	1341
251	138	− 009	1109	263 5723	+ 031	1856
252	4463	− 2115	12119	264 1355	− 006	3225
253	045	− 013	316	270 608	− 061	1853
254	020	+ 003	0114	271 1104	+ 010	1233
255	0119	+ 0114	123	272 0158	+ 016	0155

Plane II.

	II	III	IIIX	II	III	μN
272	487	−072	1261	290	—	—
274	—	—	—	291	012	−000	024
275	610	+ 020	1348	292	207	+ 133	529
276	1854	−180	5116	293	—	—	—
277	215	−013	665	294	078	+ 008	304
278	219	+ 073	541	295	088	−009	237
279	878	+ 161	4024	296	—	—	—
281	035	+ 008	094	297	321	+ 009	1158
282	014	+ 003	033				
283	035	+ 001	070				
283	1488	− 397	6823				
284	072	−027	191				
285	—	—	—				
286	333	+ 002	1095				
287	1312	+ 076	3104				
288	222	+ 023	636				
289	532	−110	1344				

Plane IV

	u	u'	u''	v	v'	v''
1	349.042	1132.421	3743.340	13	·	—
2	200.086	−205.457	490.411	14	6.617	0.014	15.418
3	60.340	128.493	174.241	15	3.365	0.031	47.341
4	395.882	+103.814	864.200	16	226.417	11.474	51.324
5	6.610	1.357	15.874	17	—	—	—
6	41.887	+29.353	117.259	18	3.845	1.079	9.325
7	—	—	—	19	7.303	0.014	21.870
8	—	·	·	20	2.006	0.133	4.367
9	—	—	—	21	—	—	·
10	41.365	−7.442	115.211	22	—	—	·
11	13.241	12.666	33.087	23	9.730	3.876	25.882
12	—	·	·	24	—	—	—
13	—	—	—	25	4.540	0.425	11.717
14	9.237	12.772	20.224	26	8.063	3.885	26.829
15	—			27	·	·	·
16	18.230	4.307	43.080	28	1.289	−0.051	4.268
17	3.840	0.853	8.716	29	1.064	0.109	2.675

Plane IV.

	u	u_1	u_x		μ	μ_1	μ_x
30	1.080	0.801	33.14	60	15.818	6.317	169.318
31	.887	0.533	31.00	63	.943	.213	7.592
32	5.224	.0.315	15.564	64	1.465	− 1.253	6.187
33	1.506	−.512	4.197	65	3.817	− 1.193	11.932
34	25.434	11.171	76.772	66	643	.060	1.515
40	—		—	67	11.613	1.912	40.454
41	4.523	.135	9.118	68	—	—	
42	2.598	.789	6.431	69	—	—	—
43	1.849	.117	3.543	62	876	.042	2.163
44	5.764	.883	13.910	61	5.228	− 4.499	16.472
45	3.567	.408	5.897	63	1189	.171	4.464
46	2.480	.155	8.070	63	4094	−.776	9.076
47	2.845	− 322	8.334	64	4399	−.261	11.476
48	7.028	1.115	22.746	65	9866	1.490	31.413
49	6.414	− 1.315	24.264	66	446	− 077	1371
52	947	.041	3.155	67	832	.026	1986
51	4.232	.930	9.408	63	6511	.722	20.134

Plane IV

	u	u'	u''		u	u'	u''
y	4.163	1.165	11.264	81	1.064	.185	4.019
x	2.017	3.218	5.110	82	3.014	1.838	13.778
1	0.210	−1.788	6.782	83	3.335	−.278	9.169
2	.514	.004	1.169	84	6.761	−4.954	17.430
3	.514	−.023	1.418	90	2.501	.203	7.706
91	1.150	.058	3.466	91	4.954	−1.66	13.590
92	.445	−.128	2.730	92	4.861	4.194	25.706
93	2.876	−.041	10.408	93	3.128	−5.111	7.961
94	1.914	−.115	5.705	94	4.763	−.999	16.230
95	2.468	−.229	7.358	95	4.039	−1.814	14.128
96	2.513	.036	7.447	96	2.731	−1.934	7.300
80	1.863	−.043	4.130	97	4.227	2.083	13.197
81	1.303	−.286	4.431	98	.905	−.341	2.127
2	.384	−.007	2.212	99	—	—	—
82	.662	−.006	1.480	00	1.703	.571	5.163
83	.600	−.243	1.663	1	2.857	−.997	7.673
85	2.470	1.878	6.843	2	3.64	−.221	8.29

Plate IV

	μ	μz	μx		μ	μz	μx
1.2	6.430	1.739	19.120	120	2.102	.556	6.136
1011	1.419	.056	4.919	121	8.912	3.286	33.951
1.3	.765	.222	1.513	122	3.571	1.60	10.942
1.4	5.181	.548	14.104	123	8.09	−2.62	23.50
1.5	9.247	3.152	32.374	124	4.430	.774	12.128
1.6	2.625	−.362	7.718	125	1.849	.757	4.864
1.7	.435	−.149	1.435	126	.585	−.009	1.531
1.8	5.210	.727	15.128	127	5.046	.057	13.790
1.9	1.180	.264	3.071	128	4.450	1.217	12.208
1.10	.735	−.080	1.910	129	6.319	2.416	14.341
1.11	.585	.169	1.916	130	11.711	17.054	44.897
1.12	1.810	.270	4.470	131	.143	.022	.438
1.13	2.664	−1.491	7.363	132	.3046	−.964	8.144
1.14	3.480	.311	8.758	133	1.493	−.584	4.115
1.15	2.783	−1.466	8.447	134	—	—	—
1.16	1.584	.173	4.206	135	2.217	−.165	5.705
1.17	3.337	.100	9.266	136	.316	.087	1.197

Plane IV

	μ_x	μ_y	μ_z		μ_x	μ_y	μ_z
123	1.008	1.619	6.106	146	4.412	.133	14.118
124	.364	.021	.897	147			
131	7.920	−.626	6.988	154	—	—	—
134	1.415	.201	3.708	157	—	—	—
141	1.342	−.874	4.364	158	.553	−.027	1.697
142	.187	−.015	.382	159	.991	.320	3.109
143	.340	.145	.859	160	.893	−.062	2.583
144	3.721	.728	11.891	161	3.970	.071	12.370
145	1.247	.589	3.355	162	.697	.028	1.890
146	2.048	.991	5.248	163	—	—	—
147	.754	−.053	2.436	164	1.404	1.489	4.446
148	3.508	4.533	10.579	165	4.919	−2.899	14.665
149	.031	.001	.072	166	.349	.234	1.112
150	2.240	.011	7.344	167	.201	.015	.568
151	.657	.029	1.657	168	.4410	.014	16.019
152	1.484	.203	4.847	169	.591	.081	1.958
153	2.518	.400	9.251	170	.607	−.393	1.977

Plane IV

	u	ux	u2		u	ux	u2
	1,189	.147	3480	188			
	2.325	−.858	5.711	189	6.55	.015	1413
	3.096	2.304	9.879	190	6.786	1.459	7.5089
	1.256	−.592	3143	191	1.001	.187	3118
	—	—	—	192	12.677	−3.236	36.9541
	1,833	1.440	6.817	193	—	—	—
	.447	−.025	1525	194	3.730	2.477	9.636
	.301	.009	.120	195	.610	−.082	1.758
	2.231	.674	1.205	196	13.478	3.677	41.491
	.103	.004	.269	197	.256	.089	.728
	3.187	3.051	9.996	198	1.286	−.512	3.654
	1.290	.133	3.626	199	.854	.550	2.315
	4.44	.643	1.514	200	2.874	−.965	8.507
	.893	.046	2.707	201	.712	.164	2.051
	6.188	6.583	19.783	202	.964	.3316	2.448
	5.214	−.087	1.309	203	1.021	−.109	1.956
	1.164	−.061	2.416	204	.505	.013	1.473

Plant IV

	u	u'	ux		u	u'	ux
	.287	.006	8.24	232	.379	.031	1.053
206	.644	.103	1.814	233	.150	.009	.734
207	.214	−.001	4.14	234	.740	−.117	1.911
208	.791	−.010	2.287	235	.910	.174	2.842
209	2.420	−.432	7.223	236	.131	.085	.392
210	.363	.005	1.106	237	.393	−.135	.970
211	2.879	−.536	9.663	238	—	—	—
212	1.230	−.303	11.141	239	.364	.007	1.325
213	.904	.275	2.315	240	1.711	−.560	6.818
214	.335	−.045	1.007	241	.631	−.075	1.367
215	.213	−.001	.604	242	.048	.009	.100
216	5.633	−.713	30.154	243	1.408	−.116	3.988
217	.181	.066	.503	244	.411	.258	1.090
218	1.189	.476	2.777	245	.678	.179	1.889
219	.650	.052	1.701	246	1.928	.328	6.157
220	—	—	—	247	.204	.077	.543
221	3.649	1.998	11.282	248	1.522	.454	4.786

Plane IV

	u	u'	u''		u	u'	u''
230	.061	.011	.224	240	.211	.067	.540
231	.300	.035	.947	241	.536	.011	.183
232	3.188	−.878	10.163	242	1.340	.030	2.926
233	.352	.007	.986	243	1.050	.516	3.176
234	.137	−.007	.404	244	.232	.060	.835
235	.013	.000	.031	245	.231	.030	.507
236	.159	.147	2.973	246	.026	.062	.078
237	.500	1.000	1.946	247	.163	−.001	.505
238	3404	−1.568	11.016	248	.722	.144	2.159
239	.073	.007	.171	249	.018	−.011	.030
240	.029	−.011	.051	250	1.788	−.441	4.169
241	2.379	−.141	7.864	251	.043	.008	.111
242	.150	.077	.481	252	.577	.065	1.547
243	.454	.031	1.223	253	.149	.026	.340
244	.081	.014	.234	254	.562	−.038	.131
245	.015	−.000	.035	255	.414	−.070	1.400
246	.050	.006	.130	256	.019	.002	.221

Plane IV

233	396	1 411	8 37	240	—	—	—
234	— —	—	-	241	.013	001	.018
235	.646	1 1 8	1564	242	307	.0 32	.5 2 9
236	2.070	- 0 73	6416	243	—	—	-
237	.2 16	- 005	.6 77	244	064	.021	.201
238	.2 11	.037	505	245	.103	- 011	.3 19
239	.821	.104	3.520	246	—	- —	-
250	039	-.005	.120	247	.258	-104	.937
251	.017	- 000	.041				
252	.038	013	091				
253	1.455	.9 2 9	6.599				
254	.055	-.007	.175				
255	—	- —	-				
256	.320	.2 80	.993				
257	1.268	.475	2 402				
258	258	.043	.3 66				
259	557	102	1.894				

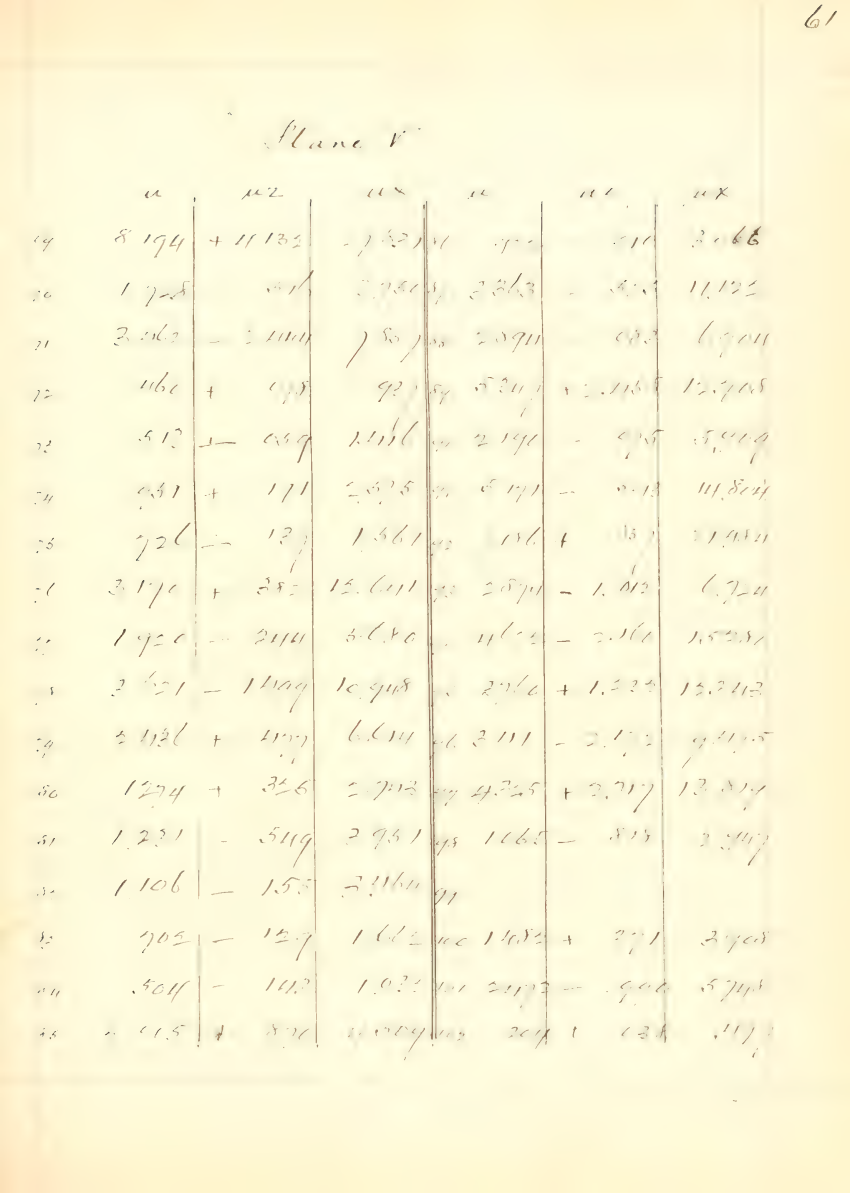
Plane V

	u	u_2	u_x		v	v_1	v_x
1	[illegible]	− 33.4117	1 091.575	18			
2	395.290	+ 127.510	621.918	19	1.061	+ 0.361	1.573
3	57.701	+ 40.605	171.733	20	1.736	+ 0.[illegible]9	4.853
4	391.536	+ 24.072	1 [illegible]3.300	21			
5	8.032	+ 1.674	22.911	22	2.[illegible]46	− 0.072	6.[illegible]6
6	36.091	+ 17.337	87.262	23	1.[illegible]58	− 1.[illegible]03	1.50244
7				24	5.256	+ 0.311	2.[illegible]62
8				25	1.5465	+ 0.885	2.713
9				26			
10	111.996	− 2.121	117.622	27			
11	12.061	+ 1.285	27.155	28	1.[illegible]21	+ [illegible]	[illegible]
12				29			
13				30	1.1[illegible]2	− 0.231	1.0234
14	10.652	− 0.327	2[illegible]8.96	31	8.554	− 12.906	20.410
15				32			
16	25.423	+ 3.525	[illegible]4.405	33	0.906	− 0.070	2.116
17	3.729	+ 0.455	8.021	34	1.[illegible]2	+ [illegible]	3.301

Plane V.

	u	u_2	u_x	u	u_1	u_x
35	0.092	− 0.4165	2.832	1.1121	1.1942	59.294
36	.988	− .762	3.257	1.0800	2.146	3.340
37	6.256	− 5.8	16.362	2.1117	.616	11.691
38	1.694	− .2854	2.826	3.3321	− 1.158	9.306
39	22.847	+ 11.420	61.776	.529	+ 1.113	1.096
40				1.1023	+ 9.329	31.229
41	11.920	+ 2.33	9.931			
42	2.018	− 0.266	11.091			
43	1.784	+ .077	2.2264	1.004	+ 173	2.319
44	6.416	+ .991	18.109	1.4832	− 2.3[illegible]	12.333
45	3.113	× .594	0.123	1.005	+ 0.85	3.913
46	2.368	+ 2.24	5.556	3.853	− .755	8.034
47	2.102	− .516	6.1115	1.1465	− 1.21	11.630
48	4.113	+ 2.652	13.331	9.560	+ 1.641	29.195
49	56.80	− 3.[illegible]	19.528	1.128	− 0.[illegible]	1.[illegible]
50	.761	+ 0.45	1.948	1.905	+ 0.64	1.1241
51	11.117	+ 1.818	10.381	5.363	− 1.352	13.353

Plane V



Plane V

	u		u2	ux	u	u1	u4
103	6.211	+	1.121	16.209	130	2.135	.711	6.325
104	1.428	−	.181	5.611	131	.9925	.3 111	26.758
105	.11	+	.3 111	1.32 9	132	2.684	+ .33	11.646
106	11.611	−	.582	16.1134	133	.321	− .195	2.1181
107	9.6611	−	1.123	38.212	134	1.886	+ .62	11.90
108	2.5841	−	.909	9.3111	135	1.849	+ .291	11.649
109	.911	−	.323	2.312	136	.229	− .189	1.212
110	11.599	−	.726	12.180	137	.5311	− .1393	1.5.213
111	1.287	−	.189	3.6581	138	.1681	− .321	12.615
112	.641	−	.135	1.117	139	6.211	− 2.298	14.1109
113	.926	+	.102	2.139	140	11.081	− 11.1111	22.321
114	2.711	+	.1177	5.198	141	.198	− .017	.1138
115	2.528	−	.1119	1.620	142			
116	11.012	−	.312	11.581	143	2.011	− .9 1	2.956
117	2.315	−	2.107	8.0111	144	1.588	− .526	11.192
118	1.6[illegible]	−	.123	4.1.2	145	1.819	− .149	3.836
119	3.119	+	.1123	8.192	146	.911	+ .163	.911

Plane V

	u		u'	ux		u	u'	ux
134	1.630	+	8.9	14.0710	154	5603	−4.562	4.5010
138	30.7	−	0.20	6.24	155			
139	2.392	−	1.82.5	9.112	156			
140	1.201	+	0.13	2.63.3	157			
141	1.177	−	2.01	3.124	158	8.24	− 0.1.7	1.665
142	189	−	0.16	1.11.2	159	10.71	+ 2.23	3.628
143	2.116	−	1.62	8.91	161	857	− 1.69	2.5461
144	3.0018	−	1.50	7.792	162	8.611	− 2.12	1.802
145	1.307	−	5.17	11.5.18	163	5.32	− 3.28	2.720
146	2.168	−	1.20	5.601	164			
147	710	+	0.27	2.345	165	1013	− 3.26	2.317
148	2.28.1	+	3.819	9.2.80	166	11.911	− 1.226	13.920
149	0.20	+	0.0.0	0.19.11	167	3.111	+ 1.21	3.97
150	1.918	×	1.89	5.3.24	168	1.93	+ 0.20	6.38
151	1.18	−	1.51	1.966	169	11.176	− 9.99	11.3.02
152	1.5.12	−	8.21	5.0.32	170	8.17	− 1.0	1.1.11
153	2.26.1	+	6.54	9.2.8.6	171	6.46	− 2.31	1.9.36

Plane F

	u	u'	ux	u	u'	ux
11	1.335	+ .037	1.3391	188		
12	2.591	− .711	1.8801	189	.5111	+ .1110	1.624
13	2.926	+ 1.917	8.028	190	1.7118	+ 3.4138	32.151
130	1.120	− .5115	2.962	191	.9111	+ .552	2.711
135				192	1.0162	− 2.787	33.935
14	1.518	+ 1.554	1.6516	193			
15	3.85	− .227	1.1661	194	2.961	+ 1.385	1.6164
153	2.06	− .015	.629	195	1.25	− .225	1.8111
152	2.013	+ .221	.5811	196	13.529	− 1.512	111.035
154	1.21	− .041	2.1	197	.520	− .012	.588
151	3.308	+ 3.326	13.883	198	1.037	+ .110	23.211
132	1.2411	+ .238	3.1129	199	.828	− .1111	2.1111
133	1.110	+ .501	1.1161	200	2.339	− .856	52.991
134	.936	− .039	2.061	201	.594	+ .132	1.1427
135	1.045	+ 1.005	1.5320	202	3.1185	+ 3.157	3.1206
136	1.1115	− .510	.928	203	1.013	− 1.111	2.1163
139	1.1355	− .1191	2.329	204	1.1191	+ 1.321	2.0361

Plane V

	u		u'	ux		u		u'	ux
205	278	+	138	7.5	212	314	−	116	9[illegible]
206	111	+	102	1 038	213	296	−	032	9[illegible]
207	216	−	028	1032	214	72[illegible]	−	138	13[illegible]
208	1003		0285	2 383	215	201	+	0963	16 91
209	2 315		8[illegible]	6 915	216	117	+	0[illegible]	2 81
210	2118	−	079	1018	217	1116	−	1,8	1 [illegible]3
211	3,686	−	060	10 694	218				
212	1,[illegible]22	−	020	4 [illegible]11	219	1[illegible]95		258	48
213	832	+	1286	1 989	220	2[illegible]3	+	6201	6 [illegible]13
214	396	−	021	1 054	221	582	−	1111	12 91
215	210		0[illegible]5	631	222	0,53	+	0[illegible]2	12 2
216	7,315	+	11[illegible]61	21 650	223	1 [illegible]4	+	232	3 [illegible]19
217	136	+	030	286	224	3[illegible]5	+	1[illegible]3	6 [illegible]11
218	1,162	+	2[illegible]13	2 768	225	26[illegible]		1111	13 91
219	1809	+	136	1 [illegible]13	226	1 [illegible]91	+	586	21 [illegible]4
220					227	19[illegible]		0[illegible]3	3 [illegible]21
221	2 312	+	1[illegible]15	9 [illegible]301	228	1 [illegible]06	−	1[illegible]9	11 [illegible]23

Plane V

	u		u2	uX	u		u2	uX
239	0,262	+	0,115	1,1	0,211	+	1,2	0,5
240	0,91	+	0,411	8,87	0,9	0,10	-	0,1	1,81
241		-	0,211	8,999	2,8	1,88	+	0,2	0,1107
242	0,89	+	0,12	1,205	0,9	1,11	-	0,1	3,267
243	1,39	-	0,0	1,111	0,80	0,0	+	0,72	1,65
244	0,11	+	0,01	0,25	0,31	0,34	+	0,11	1,08
245	1,98	-	0,9	2,841	0,2	0,1	-	0,0	0,11
246	0,9	+	0,3	1,791	0,3	1,13	+	1,1	1,11
247	3,035	-	1,13	3,791	0,4	1,60	-	0,2	1,913
248	0,73	+	0,1	1,68	0,15	0,19	-	0,11	0,32
249	0,11		0,19	0,8	0,6	1,155	+	1,111	2,345
250	2,494	-	1,792	8,64	0,7	0,11	-	0,13	1,03
251	1,25	+	0,21	1,59	0,8	0,32	-	0,12	1,908
252	1,11	+	0,16	1,271	0,6	1,35	+	0,21	2,17
253	0,12	+	0,11	1,31	0,5	1,03	-	0,11	1,96
254	0,17	-	0,02	0,21	0,9	3,99	-	0,13	0,23
255	0,31	-	0,11	1,11	0,13	0,20	+	0,11	1,31

Plane V

	u	u1	ux	μ	μ2	ux
273	368	+ 226	681	290		
274				291 014	+ 001	241
275	163	+ 132	2200	292 195	− 196	1191
276	1080	+ 219	6491	293		
277	200	+ 111	5911	294 084	+ 010	126
278	129	− 028	495	295 104	− 012	332
279	763	− 032	3923	296		
280	012	− 018	125	297 251	− 084	907
281	016	− 003	028			
282	034	+ 013	094			
283	1705	− 1455	5016			
284	019	+ 005	011			
285						
286	317	+ 272	972			
287	1263	+ 1101	3881			
288	291	+ 024	608			
289	113	+ 102	1308			

Plane VI

	u	u_2	u_x		μ	μ_2	μ_x
1	390.817	−201.186	114;026	18			
2	263.539	+228.616	8.58920	19	.5171	+0.271	11430
3	115.694	+ 9.118	111.158	20	15838	−0.1101	11036
4	415.1130	−86.516	951.620	21			
5	8.537	− 0362	25982	22	22663	−15938	65657
6	30.432	+ 3209	62003	23	4907	− 2712	15183
7				24	8761	− 0396	31041
8				25	1518	+ 0930	2607
9				26			
10	46283	+ 6716	1141233	27			
11	11724	− 8787	25939	28	11436	−0.3511	26091
12				29			
13				30	3603	+0.108	29916
14	12 289	− 5178	3.5805	31	7606	−10.1711	23877
15				32			
16	22 1163	+0025	.58 871	33	0...	−0031	14111
17	4 013	−0382	9290	34	1259	+0204	37112

Plane 17

	μ	μ2	ux	n	u1	ux
35	2.944	- 0114	1.818	16.230	2.833	5.5191
36	551	- 218	1.335	991	- 023	2.144
37	4.556	- 1101	11.945	21.191	+ 258	11.881
38	1.608	+ 3211	11.952	24.691	- 549	- 2.211
39	21.026	+ 1220	5.4864	648	+ 110	1.122
40				4.877	+ 6.211	29.819
41	6.227	+ 238	2.514			
42	1.912	- 211	3.210			
43	1.932	+ 235	3.961	930	+ 102	2.126
44	6.281	- 346	16.500	11.138	- 2.113	10.319
45	3.915	- 013	10.018	881	- 024	2.454
46	2.044	+ 063	4.814	1.086	- 126	9.112
47	24.128	- 299	6.911	25.037	+ 11.5	14.989
48	6.845	+ 1.396	21.559	11.191	+ 219	33.615
49	11.525	+ 112.8	12.076	2.16	- 031	1.123
50	595	+ 023	1.194	6.53	+ 11.5	13.216
51	11.962	+ 910	11.911	11.951	- 1.111	10.2111

Plane VI

	u	μz	ux		u	u_1	ux
69	8.013	+ 2.367	26.613	86	.710	— .111	1.148
70	1.[illegible]7	— .237	3.656	87	2.116	— 1.[illegible]	7.34
71	11.201	+ 1.223	13.11[illegible]	88	2.8[illegible]	+ .11[illegible]	6.38[illegible]
72	1.153	+ .073	9.15	89	4.724	+ .90[illegible]	7.9[illegible]
73	4.911	— .031	1.215	90	2.151	[illegible]	[illegible]
74	7.76	+ .084	1.916	91	1.811	— .21[illegible]	12.808
75	6.29	— .061	1.172	92	6.842	— 2.501	19.110
76	2.880	+ .350	10.432	93	3.047	— .652	2.559
77	1.647	— .101	4.124	94	11.242	— 1.052	13.178
78	3.684	— .119	11.234	95	3.169	+ .1492	6.988
79	2.008	+ .202	11.494	96	3.526	+ .1136	12.166
80	1.0[illegible]1	+ .285	2.022	97	3.457	+ .3412	8.825
81	1.[illegible]2	— .195	2.620	98	1.222	— .1115	3.883
82	1.1[illegible]	— .163	9.273	99	1.1163	— .170	3.810
83	1.[illegible]2	— .1141	1.[illegible]9	100	2.21[illegible]	— .[illegible]	[illegible]
84	.1116	+ .006	.808	101	1.[illegible]0	+ .027	3.145
85	1.31[illegible]	+ .[illegible]1[illegible]	3.974	102	6.256	— .356	15.731

Plane II

	u		u_1	u_2	u		u_1	u_2
103	6.286	−	.206	13.321	[illegible]	−	1.91	.196
104	1.25	−	1.38	3.84	[illegible]		.46	21.304
105	.552	+	.454	1.21	[illegible]	+	1.2	13.800
106	3.912		.332	10.951	[illegible]	+	.285	2.056
107	9.659	+	.681	28.325	[illegible]	+	.426	13.400
108	3.112	−	.312	11.664	[illegible]	+	1.28	.4768
109	.519	−	.093	1.235	[illegible]	−	.0114	1.071
111	11.662	−	1.312	11.681	[illegible]		1.965	15.381
111	1.263	+	1.21	.2516	[illegible]	−	.1011	.0320
112	.601	+	.206	1.271	[illegible]		.946	2.607
113	1.030	−	1.01	2.0145	[illegible]	−	.713	2.4667
1111	2.260	+	1.47	6.824	[illegible]		.145	.522
115	2.085	+	.321	1.1512				
111	1.355	−	.561	13.117	[illegible]	+	1.911	9.913
117	2.512	−	.1.1	8.093	[illegible]		.215	3.811
111	1.1.2		1.491	3.652	[illegible]	−	.038	2.1115
119	2.818	+	.561	[illegible]	[illegible]	+	1.491	[illegible]

Plane VI

	m	m2	mx	m	m2	mx
13	9.571	+ 734	3836	154	5347	− 5965	16830
134	204	− 025	629	155			
135	3916	+ 2052	2567	156			
136	1202	+ 100	2623	157			
137	7[illegible]	− 0[illegible]	2[illegible]85	158	528	+ 020	1512
138	211	+ 013	552	159	1.0116	− 196	31161
139	577	− 011	1055	160	836	− 096	52365
140	2.435	− 111	5187	161	8148	− 209	1936
141	1.169	− 921	4392	162	913	− 226	32329
142	2.259	− 1291	6085	163			
143	717	+ 072	7205	164	7221	− 011	11571
144	2.895	− 291	7204	165	14905	+1360	14506
145	0[illegible]7	+ 0[illegible]0	161	166	255	− 044	2[illegible]75
146	1.[illegible]98	+ 111[illegible]	2[illegible]00	167	1[illegible]6	+ 0[illegible]1	3445
147	196	− 196	1856	168	8[illegible]12	+ 851	12384
148	1.132	− 925	11513	169	1142	− 011	1[illegible]07
149	2349	+ 1089	8050	170	630	+ 203	16552

Plane VI

	u	u'	uu	u	u'	uu
131	1,1432	− 165	4,9841	193		
132	1,6497	− 025	4,2132	194	521	+ [illegible]
133	2,212	− 031	3,0411	195	6,1211	+ 1,2445
134	1,362	+ 016	3,694	196	8,9	+ 0,82
135				197	3,228	− 1,33
136	3,291	+ 1,156	3,565	198		
137	3,5	− 006	702	199	2,811	+ 312
138	303	− 024	732	200	620	− 141
139	1,832	+ 641	4,815	201	3,189	− 4,993
140	133	+ 102	3,54	202	206	063
141	2,140	− 353	11,684	203	867	+ 239
142	15,31	− 062	24,104	204	890	− 600
143	298	− 041	669	205	24,575	+ 064
144	1,01	− 007	3,421	206	5,45	+ 015
145	5,911	+ 4,025	41,237	207	86,941	− 9,9
146	1,241	− 138	1,39	208	9,16	+ 0,45
147	1,591	− 676	4,510	209	533	+ 157

Plate VI

	u	u'	u''	u	u'	u''
205	2649	+ 1240	132	201 4444	− 0112	11.3
206	6016	− 0110	1882	202 46	− 030	592
207	22	− 0349	311	203 2711	0.3	1,944
208	5041	− 0444	2.3.1	204 316	+ 5.15	1 611
209	2.1425	− 0101	1,488	205 1211	− 0.01	592
210	209	− 011	193	206 5574	− 0005	1,916.5
211	2,1091	+ 5049	3336	207		
212	1102	+ 0145	2.211	208 5978	− 2.80	1.8
213	816	− 113	2,123	209 [illegible]	+ 906	3.4182
214	3043	− 0111	1,532	210 2824	− 0085	1,8.24
215	201	− 013	236	211 012	+ 001	176
216	5,637	+ 2782	13.411	212 1185	+ 362	2.326
217	130	+ 011	2.84	213 2445	− 013	523
218	1,355	+ 332	2.1241	214 601	− 2492	1.3.4
219	1.37	+ 121	811	215 1,382	+ 293	3.1.3
220				216 1997	− 0051	3.16
221	2.1411	− 216	8.642	217 1330	+ 1152	3.15.5

Plane II

	u	u2	ux	u	u2	u2
201	042	+ 006	101	256	[illegible]	[illegible]
210	336	- 012	588	[illegible]	[illegible]	[illegible]
211	2.914	+ 600	2116	253 1851	+ 1156	5[illegible]
212	3.91	+ 207	1223	254 1343	— [illegible]	3 886
213	134	— 000	35	256 148	+ 021	606
214	011	+ 001	0114	261 534	— 035	636
215	3.932	— 134	1822	262 021	- 006	051
216	350	+ 134	2112	263 1113	+ 003	384
217	2.338	— 1.513	6334	264 3193	— 261	11140
218	036	+ 013	186	265 006	- 002	064
219	020	— 001	1284	266 7191	+ 3633	23920
250	2.2444	— 1116	9128	267 6413	+ 012	1144
251	1113	+ 011	1116	271 7357	+ 0110	24333
252	384	+ 134	1122	272 1141	+ 0041	338
253	035	+ 006	1063	273 4114	+ 002	3.34
254	013	— 003	0534	274 341	— 004	10111
255	054	— 020	1614	275 1394	— 116	9241

Planc VI

	u	Δu	μu		u	Δu	μu
223	.2[illegible]	+ .0113	.3[illegible]	2[illegible]			
224				241	.111[illegible]	+ .0[illegible]	.0[illegible]
225	[illegible]	− .060	2.781	242	1[illegible]	− 11.3	.1113
226	2.[illegible]	+ .225	6.1153	243			
227	1.8[illegible]	+ .01[illegible]	11.1	244	.111[illegible]	− .00[illegible]	1.[illegible]
228	.2[illegible]	− .10[illegible]	.[illegible]	245	.[illegible]	[illegible]	2.111
229	.[illegible]	− .11[illegible]	2.[illegible]	246			
250	.111[illegible]	− .011	.1[illegible]	247	2410	− .00[illegible]	1.[illegible]
251	.111[illegible]	+ .00[illegible]	.[illegible]				
252	.[illegible]	− .001	.[illegible]				
253	1.[illegible]	+ .[illegible]	3.[illegible]				
254	.[illegible]	+ .[illegible]	.[illegible]				
255							
256	.[illegible]	− .00[illegible]	10[illegible]				
257	1.[illegible]	− .[illegible]	3.[illegible]				
258	.[illegible]	− .[illegible]	1.[illegible]				
259	.[illegible]	− .05[illegible]	.[illegible]				

These numbers are,

	I	II	III	IV	V	VI
$\Sigma\mu$	1825.405 ;	1110.391 ;	1143.055 ;	1172.764 ;	1801.007 ;	1442.166.
$\Sigma\mu x$	4656.382 ;	4757.641 ;	4532.393 ;	4738.380 ;	4870.300 ;	5038.626.
$\Sigma\mu z$	−564.184 ;	−237.318 ;	+174.199 ;	+580.373 ;	+509.362 ;	−77.729.

The centre of gravity is given by,

$$x_c = \frac{\Sigma\mu x}{\Sigma\mu}$$

$$z_c = \frac{\Sigma\mu z}{\Sigma\mu}$$

These are,

	Plane I	Plane II	Plane III	Plane IV	Plane V	Plane VI
x_c	2.550	2.782	2.600	2.671	2.703	2.607
z_c	−0.309	−0.134	+0.100	+0.327	+0.283	−0.054

To determine the orbit of the mean planet I have referred the centres of gravity to a rectangular system of axes, with the sun for origin. The axis of x passes

through the Vernal equinox, the axis of η, through a point in the ecliptic whose longitude is 90° and the axis of z through the pole of the ecliptic. If ξ_n, η_n, ζ_n are the coordinates of any centre of gravity in this system, and n the number of the plane in which it lies, then,

$$\xi_n = \frac{\Sigma \mu x}{\Sigma \mu} \cos(30° + (n-1)60°)$$

$$\eta_n = \frac{\Sigma \mu x}{\Sigma \mu} \sin(30° + (n-1)60°)$$

$$\zeta_n = \frac{\Sigma \mu z}{\Sigma \mu}$$

From which I find,

	I	II	III	IV	V	VI
$\log \xi$	0.34410	$-\infty$	n 0.35755	n 0.36413	$-\infty$	0.35311
$\log \eta$	0.10554	0.44424	0.11349	n 0.11367	n 0.43180	n 0.11515
$\log \zeta$	n. 9.49038	1. 9.14223	8.99974	9.51506	9.45127	n 8.70389

To find the plane which will most nearly pass through these six points, I have used the equation

$$\xi \sin\Omega \sin i - \eta \cos\Omega \sin i + \zeta \cos i = 0$$

or $$\xi u - \eta v + \zeta t = 0$$

The values of ξ, η and ζ were substituted in this equation which gave six equations of condition. I gave each equation of condition a weight proportional to the sum of the mass of the elements at the centre of gravity from the condinates of which it was formed. In this way I obtained the following normal equations for $\frac{u}{t}$, and $\frac{v}{t}$.

$$37.085\,u + 0.172\,v - 3.203\,t = 0$$

$$-0.172\,u - 38.158\,v - 1.940\,t = 0$$

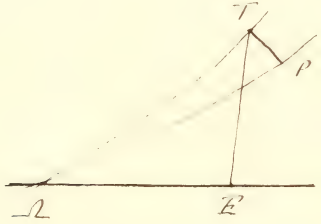
$$-3.203\,u + 1.940\,v + 0.566\,t = 0$$

The solution of these equations gives

$\frac{u}{v} = \tan\Omega = n[0.236773]$ whence $\Omega = 120^\circ 6' 14''$

and from the normal equation for t, $i = 8^\circ 35' 5''$

The next step is to find an ellipse which passes as nearly as possible through the projections of the centres of gravity on the plane just found.



Let ΩE be the ecliptic, ΩP the plane just found and, T a centre of gravity, P its orthogonal projection on the plane ΩP, also $\Omega E = L$, $E T = B$, $\Omega T = \lambda$, say, $PT = \beta$ $\Omega P = \omega + \nu$, $E \Omega P = i$, $E \Omega T = \varepsilon$, we have, knowing L and B;

$$
\begin{aligned}
\cos\lambda &= \cos B \cos(L-\Omega) \\
\sin\lambda \sin\varepsilon &= \sin B \\
\sin\lambda \cos\varepsilon &= \cos B \sin(L-\Omega) \\
\sin\beta &= \sin\lambda \sin(\varepsilon-i) \\
\cos\beta \sin(w+v) &= \sin\lambda \cos(\varepsilon-i) \\
\cos\beta \cos(w+v) &= \cos\lambda .
\end{aligned}
\qquad (A)
$$

If ρ is the distance of a centre of gravity from the sun, and r, its projection on the plane ΩP, we have,

$$\rho^2 = \left(\frac{\Sigma \mu x}{\Sigma \mu}\right)^2 + \left(\frac{\Sigma \mu z}{\Sigma \mu}\right)^2$$

$$r = \rho \cos\beta .$$

Equations (A) give β and $(w+v)$, ~~from~~ each centre of gravity, ρ is formed from x and z, and r obtained.

knowing the r's, I have used the equation

$$r = \frac{a\cos^2\varphi}{1 + \sin\varphi\cos v},$$

to determine a, φ and v.

If we put preliminarily,

$$v_{n+3} = 180° + v_n$$

we find

$$(1) \qquad a\cos^2\varphi = \frac{6}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \frac{1}{r_5} + \frac{1}{r_6}}$$

$$(2) \qquad 2\sin\varphi\cos v_1 = a\cos^2\varphi\left(\frac{1}{r_1} - \frac{1}{r_4}\right)$$

$$2\sin\varphi\cos[(v_3 - v_1) + v_1] = a\cos^2\varphi\left(\frac{1}{r_3} - \frac{1}{r_6}\right)$$

Dividing this last equation by (1)

$$(3) \qquad \cos(v_3 - v_1) - \sin(v_3 - v_1)\tan v_1 = \frac{\frac{1}{r_3} - \frac{1}{r_6}}{\frac{1}{r_1} - \frac{1}{r_4}}$$

knowing $(W+V_3)$ and $(W+V_1)$, (V_3-V_1) is known which substituted in equation (3 gives V_1. Equation (1) then gives φ, and ω is found from (1).

With these quantities I computed the r's and found the errors, in

	dr_1
I	-0.0246
II	$+0.1840$
III	-0.0565
IV	-0.0324
V	-0.0074
VI	-0.0560

For each r I formed the equation of condition

$$(1+\sin\varphi\cos V)\,dr + (r\cos V + a\sin\varphi)\cos\varphi\,d\varphi - r\sin\varphi\sin V\,dV - \cos^2\varphi\,da = 0$$

To each of which I gave a weight as in the former case, proportional to $E\rho$.

the normal equations were then formed, and ν_1 eliminated from the normal equations in ν (N) and (N+1). The resulting Normals in $d\varphi$ and da were then added and solved in the ordinary manner. By this means I found,

$$\varphi = 1^\circ\ 34'\ 28.''88$$
$$\nu_1 = -31^\circ\ 7'\ 20.''40$$
$$\log a = 0.42494$$

$$d\varphi = +\ 1'\ 56.''00$$
$$d\nu_1 = -\ 45'.28$$
$$d\log a = 0.00736$$

whence the elements of the mean planet are,

ω	$301^\circ\ 46'.52$	Mean Eq. 1890.0
Ω	$120^\circ\ 6'.14$	
i	$8^\circ\ 35'.8$	
φ	$1^\circ\ 36'.48$	
$\log a$	0.43230	

The mass of this planet would be about four and one half times that of Vesta.

To judge of the exactness of the method I have computed the coordinates of the intersections of the orbit just obtained with the original six planes.

If x_c z_c are the coordinates of the centre of gravity lying in any plane, and x_m z_m the coordinates of the intersection of the mean orbit with that plane, I have found;

Plane	x_c	x_m	Diff.	z_c	z_m	Diff
I	2.550	2.612	−.062	−.304	−.395	−.086
II	2.782	2.632	+.150	−.134	−.199	−.060
III	2.600	2.694	−.094	+.100	+.161	−.061
IV	2.671	2.801	−.130	+.327	+.423	−.096
V	2.703	2.770	−.067	+2.83	+2.10	+.073
VI	2.607	2.698	−.091	−.057	−.162	+.112

The orbit I have found differs very materially from the one derived by Svedstrup, the major axis is larger, which is partly caused, at least by the omission of several of the earlier discovered planets, where masses are relatively large, and where major axes on the contrary relatively small. I have also considered a greater number of the planets. The inclination I have found lies between those of Ceres and Vesta, and the node is nearer the node of Jupiter than the one found by Svedstrup.

Section, Third.

The Annual Variations.

Referred to the mean equinox and ecliptic 1850.0
I have found,

	Mean Orbit of Asteroids		Orbit of Mars
ϖ'	301° 16.52	ϖ	285° 15.08
Ω'	120° 6.14	Ω	51° 3.43
i'	8° 35.3	i	1° 51.11
φ'	1° 36.48	φ	5° 21.21
$\log a'$	0.43230	$\log a$	0.18289

From which are derived,

J = mutual inclination = 8° 2'.26
Ω = Ω of Asteroids on Mars = 12° 20'.42
Ω' = υ " Mars on Asteroids = 81° 13'.32

With these quantities I have computed
the perturbative function for the action of the

Asteroids on Mars, to the second powers of the inclinations and eccentricities. I have thus found, following the notation of Leverrier;

$$R_1 = \tfrac{1}{2} A_0^{(0)} + \tfrac{1}{2}(A_1^{(0)} + A_2^{(0)})(e^2 + e'^2) + \tfrac{1}{2}(A_0^{(1)} - A_1^{(1)} - A_2^{(1)}) e e' \cos(\varpi' - \omega) - \tfrac{1}{2} b^{(0)}_{\frac{1}{2}} \eta^2.$$

$\log \alpha = 4.758059$

$b^{(0)} = 2.14504 \qquad b^{(0)}_1 = 0.48535 \qquad b^{(0)}_{\frac{1}{2}} = 0.484456$

$b^{(1)} = 0.64744 \qquad b^{(1)}_1 = 0.86197. \qquad b^{(1)}_2 = 0.88653$

$C^{(1)}_1 = 3.417233.$

Whence $a' R'_1 =$ [0.011042] + [9.38814] e^2 + [9.38814] e'^2 − [9.51636] $e e' \cos(\varpi' - \omega)$ − [9.49018] η^2

Perturbations of Mars,
by Asteroids.

$\log \mathfrak{F}$	[4.30597]	
$\log(-\mathfrak{P})$	[3.38651]	
$\log \mathfrak{G}$	[4.42566]	
$\log \mathfrak{C}$	$-\infty$	

These give for the annual variations in the elements,

$$\delta e = [3.38461]\frac{m'}{\mu}$$

$$e\,d\varpi = [4.30339]\frac{m'}{\mu}$$

$$\delta p = -[4.41560]\frac{m'}{\mu}$$

$$\delta q = [3.75672]\frac{m'}{\mu}$$

} Not correctes for term in $(p\,dq+q\,dp)$

where the coefficients are logarithms.

Equating $e\,d\varpi$ to the known excess* of the motion of the perihelion, we shall have the equation,

Secular excess $= +0''.75$ (Newcomb – Astr. Constts. p110)

$$[4.30339]\ \frac{m'}{\mu} = [7.87506]$$

or

$$\log \frac{m'}{\mu} = \bar{7}.47169$$

Neglecting the mass of Mars in μ, this equation gives the mass of the planet, which if moving in the mean orbit of the Asteroids would produce the unexplained motion in the line of abscides of the orbit of Mars.

From the above equation, approximately

m = $\frac{1}{10}$ mass of the earth

= roughly $\frac{3}{4}$ mass of Mars.

As the probable superior limit to the combined mass of the Asteroids (page 16) is only about six times the mass of Vesta, or from one one thousandth to six one thousandths of the mass of Mars, it seems evident that

the explanation of the anomaly in the motion of Mars must be sought elsewhere, for although, with our present knowledge of the albedoes and densities of the minor planets, any attempt to sum the mass of the group must necessarily be a rough approximation, the great discrepancy between the probable mass, and the theoretically required mass, would seem to settle the question finally.

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